A Least-Squares Solution to the Multiple-Aircraft Wind Estimation Problem

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Version Control Revision: 1.3
Version Control Date: 2006/12/23 21:10:01 (UTC)
RCSfile: wvecprob.tex,v
LaTeX Compilation Date: December 23, 2006

Abstract

This document presents a solution to the problem of estimating wind speed and direction from simultaneous radar observations of the course and groundspeed of multiple aircraft and knowledge of the approximate cruising airspeed of each aircraft.

1 Introduction and Overview

This problem was posted by Chad Speer to the `sci.math` newsgroup in December, 2006. The problem did not at that time receive any meaningful suggestions toward a solution.

The problem is how to [uniquely] estimate wind velocity and direction in the local area from:

- Radar observations of course and groundspeed of multiple aircraft.
- Knowledge of the cruising airspeed of each aircraft (typically obtained from VFR or IFR flightplan or clearance data filed by the pilot, or from knowledge of the model of aircraft).

This solution assumes that each observed aircraft is affected by wind at the same speed and in the same direction. This is a reasonable assumption, and will generally hold true for aircraft at the same altitude separated by perhaps 20-200 nautical miles. However, this assumption may be very flawed for aircraft at different altitudes, as the winds tend to vary greatly in magnitude and direction with altitude. Relaxing the assumption of identical wind vectors affecting all observed aircraft may be a direction for future mathematical refinement.

I (Dave Ashley) will be glad to incorporate all further open questions, mathematical results, and notes into this document. I'll also be glad to include all contributors as authors.

Any mathematical results shown to work well in practice may eventually be incorporated into algorithms used in air traffic control radar.

2 Terms and Mathematical Nomenclature

All angular measurements (the angles of vectors) are in radians. (There are 2π radians in a circle. A circle of radius 1 has a circumferance of 2π.) Radians are used because certain mathematical identities only hold with angular measurements in radians (for example, $\lim_{x \to 0} \frac{\sin x}{x} = 1$).

The *heading* of an aircraft is the direction the aircraft is pointed, whereas the *course* is the direction of the ground path of the aircraft. In the presence of wind other than a direct headwind or tailwind, the heading is unequal to the course. The heading of the aircraft is known by the pilot but not reported to anyone on the ground. The course of the aircraft is known from radar data.

All angular measurements used to specify wind direction, aircraft heading, or aircraft course are with respect to true North and proceed clockwise so 0 is true North, $\pi/2$ is true East, π is true South, and $3\pi/2$ is true West.

Canonically, and unless convenient for mathematical reasons, all angles are expressed as $\in [0, 2\pi)$.

Vectors are differentiated from scalars with an overlying arrow—v_i is a scalar but $\vec{v_i}$ is a vector.

The local wind vector is $\vec{w}$ with magnitude v_w and direction θ_w.

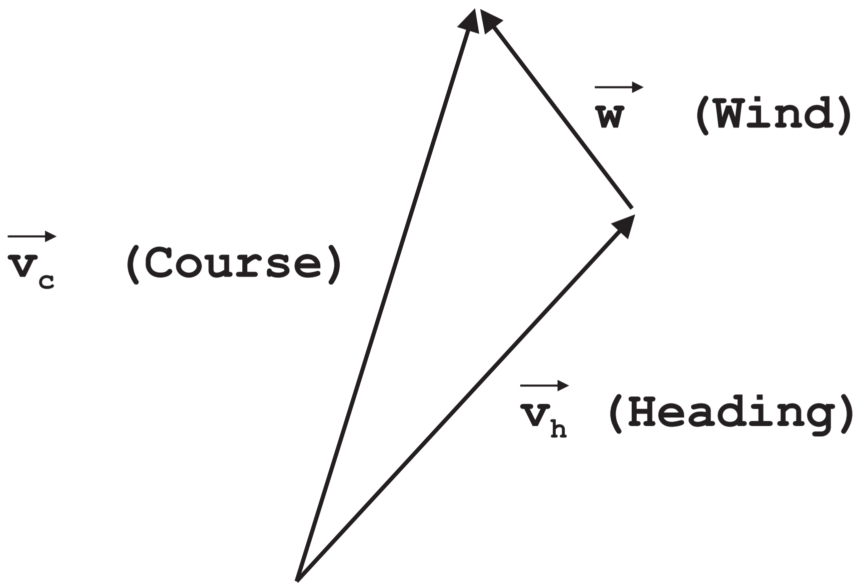

Figure 1: Wind Triangle

Each aircraft is denoted A_i, $i \in \{1, 2, \dots\}$; and has a heading vector $\vec{v_{hAi}}$ with magnitude v_{hAi} and heading direction θ_{hAi}. The course of the aircraft A_i is denoted as a vector $\vec{v_{cAi}}$ with magnitude v_{cAi} and course direction θ_{cAi}. Note that the course is observed by radar.

3 The Wind Triangle

Airplanes fly in a moving block of air, so that the aircraft's ground motion is the vector sum of the air motion with respect to the ground and the aircraft's motion with respect to the air (Fig. 1).

For each aircraft A_i,

$$\vec{v_{cAi}} = \vec{w} + \vec{v_{hAi}}. \tag{1}$$

(1) is known as the *wind triangle* because student pilots are taught to make this calculation graphically by drawing a triangle of three vectors on graph paper or by using a mechanical computer such as the E-6B[1].

To a person who has never piloted an aircraft, (1) may be unexpected. It is very common for pilots to have a course that differs from the heading by more than 10 degrees; and this is visually apparent in an airplane when tracking roads or freeways below or when landing in a crosswind.

[1] `http://en.wikipedia.org/wiki/E6B`.

4 The One-Aircraft Case

With only a single aircraft A_1,

$$\vec{v_{cA1}} = \vec{w} + \vec{v_{hA1}}. \tag{2}$$

Separating (2) into x- and y-components yields

$$v_{cA1} \cos \theta_{cA1} = w \cos \theta_w + v_{hA1} \cos \theta_{hA1} \tag{3}$$
$$v_{cA1} \sin \theta_{cA1} = w \sin \theta_w + v_{hA1} \sin \theta_{hA1} \tag{4}$$

The following quantities are known:

- v_{cA1} (from radar observation of the aircraft).
- θ_{cA1} (from radar observation of the aircraft).
- v_{hA1} (the cruising speed of the aircraft, usually filed by the pilot as part of the VFR or IFR clearance process).

The following quantities are unknown:

- w (wind velocity).
- θ_w (wind direction).
- θ_{hA1} (Note as discussed above that *heading* and *course* are distinct. The course is known from radar observation, but the heading—the direction the aircraft is pointed—is not known.[2])

With two equations and three unknowns, it would normally be expected that the solution is a set that can be parameterized with one parameter.

It can be seen graphically (Fig. 2) that an infinite number of solutions exist, parameterized by $0 \leq \theta_{hA1} < 2\pi$. A heading vector with the appropriate magnitude (v_{hA1}, the cruising speed of the aircraft) can be chosen so that its endpoint is anywhere on the circle C in Fig. 2, and a wind vector $\vec{w}$ can then be chosen to solve the equations.

TO DO:

- Need an analytic statement of the graphical solution shown in Fig. 2.
- Need to see if there are other ways to parameterize the solution set (these may become relevant for a least-squares solution).

[2]More precisely, the heading is not known by anyone *on the ground*. The heading is indicated by at least one aircraft instrument and known to the pilot, but this information is not communicated to anyone else.

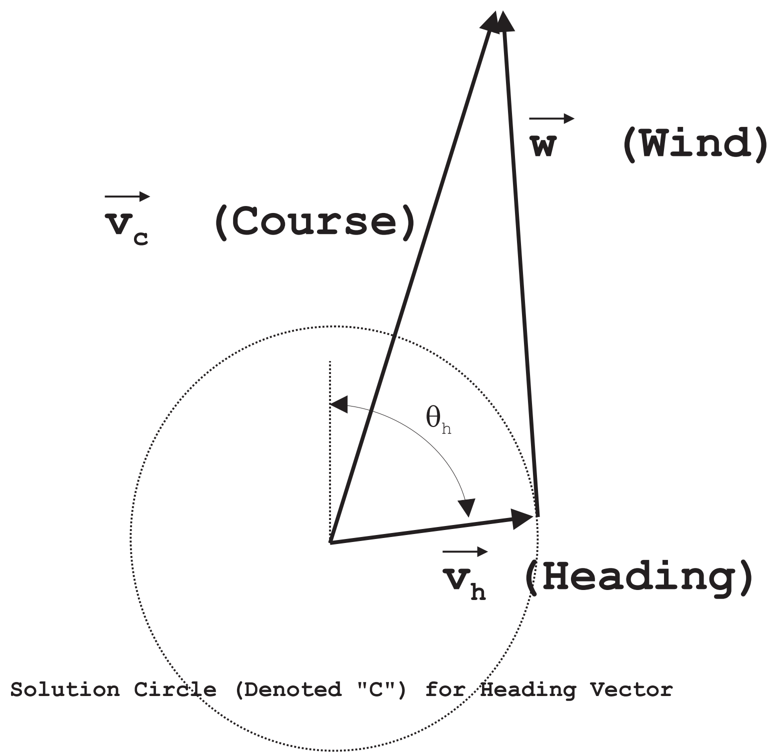

Figure 2: Graphical Solution for Single Aircraft Case

5 The Two-Aircraft Case

I expect the solution to be unique with two aircraft.
TO DO:

- Need a graphical solution.
- Need an analytic solution.

6 The Multiple Aircraft Case

I expect the solution to be overspecified.
TO DO:

- Need to set this up as a least-squares problem or in any other framework where the result is unique for a given set of input data (needs some thought because the equations aren't traditional linear algebra equations).
- Need to figure out how to adjust sensitivity to radar observation errors of aircraft course versus groundspeed. Suspect this can be done with a single coefficient.